

1⁻⁺ light exotic mesons in QCD

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We systematically re-examine the extraction of the masses and couplings of the 1⁻⁺ hybrid, four-quark and molecule mesons from QCD spectral sum rules (QSSR). To NLO for the perturbative and power corrections, the hybrid mass is $M_H = 1.81(6)$ GeV and $M_H \leq 2.2(2)$ GeV from the positivity of the spectral function. In the same way, but to LO, the four-quark state mass is $M_{4q} = 1.70(4)$ GeV and $M_{4q} \leq 2.4(1)$ GeV, while the molecule mass is about 1.3(1) GeV. The observed $\pi_1(1400)$ and $\pi_1(1600)$ might be explained by a two-component mixing with the set of input masses ($1.2 \sim 1.3$; $1.70 \sim 1.74$) GeV and with a mixing angle $\theta \simeq -(11.7 \pm 2.2)^\circ$, which slightly favours a molecule/four-quark mixing, and which eventually suggests that the $\pi_1(2015)$ is mostly an hybrid meson. Isospin and non-exotic partners of the previous states and some of their radial excitations are also expected to be found in the energy region around 2 GeV. Further tests of this phenomenological scenario are required.

1. Introduction

Light hybrid mesons with the exotic quantum number 1⁻⁺ and with characterisitc decays into $\rho\pi$, $b_1\pi$ and $\eta'\pi$ have been expected to be easily found in hadronic experiments. There are indeed two good experimental candidates $\pi_1(1400)$, $\pi_1(1600)$ [1], but their masses are lower than the lattice predictions in the quenched approximation [2] and with two dynamical quarks (2q) [3]:

$$M_H|_{\text{quenched}} \simeq 1.9 \text{ GeV}, \quad M_H|_{2q} \simeq 2.2(2) \text{ GeV}. \quad (1)$$

First studied in QCD in [4,5], lowest order (LO) results from QCD spectral sum rules (QSSR)[7] à la SVZ [8] have been obtained in [6,9,10]:

$$M_H|_{\text{LO}} \simeq (1.6 \sim 2.1) \text{ GeV}, \quad (2)$$

which, after the inclusion of radiative corrections (partly obtained in [11] and completed in [12]) lead to the upper bound and estimate in units of GeV:

$$M_H|_{\text{NLO}} \leq (1.9 \sim 2.0), \quad M_H|_{\text{NLO}} = (1.6 \sim 1.7). \quad (3)$$

Though the QSSR result tends to favour the hybrid assignement of the $\pi_1(1600)$ meson, as quoted in [1], it is important to understand the discrepancy of the above QSSR results with the ones from the lattice and to a lesser extent with the one in [13] where it is claimed that the inclusion of the operator anomalous dimension obtained in [11] decreases substantially the previous result in Eq. (3) down to 1.2 GeV, though a such effect disappear to LO, and a similar one in the tensor meson has lead to negligible corrections [14]. One can also notice that [12] use as input the value of the four-quark condensate from vacuum saturation which has been shown from different channels and different groups to be violated by a factor 2-3 [7,15,16,17,18,19]. In the following, we revisit the existing NLO results by paying attention on the influence of different QCD input parameters used in the analysis. We shall also check the effect of the operator anomalous dimension and study the new effect of

the tachyonic gluon mass introduced in [11,20,21] but overlooked in [12]. For a more robust estimate, we shall consider results both from finite energy (FESR) and Laplace (LSR) sum rules. We complement our studies by reexamining the predictions obtained for the 1⁻⁺ light four-quark [22] and molecule [23] states and conclude the paper by a phenomenological discussion on the possible nature of the observed $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$ mesons.

2. QCD spectral sum rules (QSSR)

Description of the method

Since its discovery in 1979 [8], QSSR has proved to be a powerful method for understanding the hadronic properties in terms of the fundamental QCD parameters such as the QCD coupling α_s , the (running) quark masses and the quark and/or gluon QCD vacuum condensates [7]. In practice (like also the lattice), one starts the analysis from the two-point correlator (standard notations):

$$\begin{aligned} \Pi_{V/A}^{\mu\nu}(q^2) &\equiv i \int d^4x e^{iqx} \langle 0 | \mathcal{T} \mathcal{O}_{V/A}^\mu(x) \left(\mathcal{O}_{V/A}^\nu(0) \right)^\dagger | 0 \rangle \\ &= - (g^{\mu\nu} q^2 - q^\mu q^\nu) \Pi_{V/A}^{(1)}(q^2) \\ &\quad + q^\mu q^\nu \Pi_{V/A}^{(0)}(q^2), \end{aligned} \quad (4)$$

built from the hadronic local currents $\mathcal{O}_\mu^{V/A}(x)$:

$$\begin{aligned} \mathcal{O}_V^\mu(x) &\equiv : g \bar{\psi}_i \lambda_a \gamma_\nu \psi_j G_a^{\mu\nu} : , \\ \mathcal{O}_A^\mu(x) &\equiv : g \bar{\psi}_i \lambda_a \gamma_\nu \gamma_5 \psi_j G_a^{\mu\nu} : \end{aligned} \quad (5)$$

which select the specific quantum numbers of the hybrid mesons; A and V refer respectively to the vector and axial-vector currents. The invariant $\Pi^{(1)}$ and $\Pi^{(0)}$ refer to the spin one and zero mesons. One exploits, in the sum rule approaches, the analyticity property of the correlator which obeys the well-known Källén–Lehmann dispersion relation:

$$\Pi_{V/A}^{(1,0)}(q^2) = \int_0^\infty \frac{dt}{t - q^2 - i\epsilon} \frac{1}{\pi} \text{Im} \Pi_{V/A}^{(1,0)} + \dots \quad (6)$$

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where ... represent subtraction terms which are polynomials in the q^2 -variable. In this way, the *sum rule* expresses in a clear way the *duality* between the integral involving the spectral function $\text{Im}\Pi_{V/A}^{(1,0)}(t)$ (which can be measured experimentally), and the full correlator $\Pi_{V/A}^{(1,0)}(q^2)$. The latter can be calculated directly in QCD in the Euclidean space-time using perturbation theory (provided that $-q^2 + m^2$ (m being the running quark mass) is much greater than Λ^2), and the Wilson expansion in terms of the increasing dimensions of the quark and/or gluon condensates which simulate the non-perturbative effects of QCD.

The SVZ expansion and beyond

Using the Operator Product Expansion (OPE) [8], the two-point correlator reads for $m = 0$:

$$\Pi_{V/A}^{(1,0)}(q^2) \simeq \sum_{D=0,2,\dots} \frac{1}{(q^2)^{D/2}} \sum_{\dim O=D} C(q^2, \nu) \langle \mathcal{O}(\nu) \rangle ,$$

where ν is an arbitrary scale that separates the long- and short-distance dynamics; C are the Wilson coefficients calculable in perturbative QCD by means of Feynman diagrams techniques; $\langle \mathcal{O}(\nu) \rangle$ are the quark and/or gluon condensates of dimension D . In the massless quark limit, one may expect the absence of terms of dimension 2 due to gauge invariance. However, it has been emphasized recently [21] that the resummation of the large order terms of the perturbative series can be mimiced by the effect of a tachyonic gluon mass λ which generates an extra $1/q^2$ term not present in the original OPE. This short distance mass has been estimated from the e^+e^- data [18,20] and pion sum rule [20] to be:

$$\frac{\alpha_s}{\pi} \lambda^2 \simeq -(0.07 \pm 0.03) \text{ GeV}^2. \quad (7)$$

In addition to Eq. (7), the strengths of the vacuum condensates having dimensions $D \leq 6$ are also under good control, namely:

- $\langle \alpha_s G^2 \rangle \simeq (6.8 \pm 1.3) 10^{-2} \text{ GeV}^4$ from sum rules of $e^+e^- \rightarrow I = 1$ hadrons [18] and heavy quarkonia [26,24,28];
- $g \langle \bar{\psi} G \psi \rangle \equiv g \langle \bar{\psi} \lambda_a \sigma^{\mu\nu} G_{\mu\nu}^a \psi \rangle \simeq 2 \times (0.8 \pm 0.1) \text{ GeV}^2 \langle \bar{\psi} \psi \rangle$, from the baryons [29,19] and the heavy-light mesons [30] systems;
- $\alpha_s \langle \bar{\psi} \psi \rangle^2 \simeq (4.5 \pm 0.3) \times 10^{-4} \text{ GeV}^6$ from $e^+e^- \rightarrow I = 1$ hadrons [18] and τ -decay data [17], where a deviation from the vacuum saturation estimate has been noticed from different studies [7] and [15, 16,17,18];
- $g^3 \langle G^3 \rangle \simeq (1.2 \pm 0.2) \text{ GeV}^2 \langle \alpha_s G^2 \rangle$ from dilute gaz instantons [31] and lattice calculations [32].

In the numerics, we shall use the value of the QCD scale:

$$\Lambda_3 = (353 \pm 15) \text{ MeV} , \quad (8)$$

deduced recently to 4-loops from the value of $\alpha_s(M_\tau) = 0.3249(80)$ from τ -decay [17].

Spectral function

In the absence of the complete data, the spectral function is often parametrized using the “naïve” duality ansatz:

$$\frac{1}{\pi} \text{Im}\Pi_{V/A}^{(1,0)}(t) \simeq 2M_H^4 f_H^2 \delta(t - M_H^2) + \text{“QCD continuum”} \times \theta(t - t_c) , \quad (9)$$

which has been tested [7] using e^+e^- and τ -decay data, to give a good description of the spectral integral in the sum rule analysis even in the case of the broad σ and K_0^* states [33]; f_H (an analogue to f_π) is the hadron’s coupling to the current; while t_c is the QCD continuum’s threshold.

Sum rules and optimization procedure

Among the different sum rules discussed in the literature [7], we shall be concerned with the following Laplace sum rule (LSR) and its ratio [8,25,26] :

$$\begin{aligned} \mathcal{L}_n^{(1,0)}(\tau) &= \int_0^\infty dt t^n \exp(-t\tau) \frac{1}{\pi} \text{Im}\Pi_{V/A}^{(1,0)}(t) , \\ \mathcal{R}_n(\tau) &\equiv -\frac{d}{d\tau} \log \mathcal{L}_n , \quad (n \geq 0) . \end{aligned} \quad (10)$$

The advantage of the Laplace sum rules with respect to the previous dispersion relation is the presence of the exponential weight factor which enhances the contribution of the lowest resonance and low-energy region accessible experimentally. For the QCD side, this procedure has eliminated the ambiguity carried by subtraction constants, arbitrary polynomial in q^2 , and has improved the convergence of the OPE by the presence of the factorial dumping factor for each condensates of given dimensions. The ratio of the sum rules is a useful quantity to work with, in the determination of the resonance mass, as it is equal to the meson mass squared, in the usual duality ansatz parametrization. As one can notice, there are “a priori” two free external parameters (τ, t_c) in the analysis. The optimized result will be (in principle) insensitive to their variations. In some cases, the t_c -stability is not reached due to the too naïve parametrization of the spectral function. In order to restore the t_c -stability of the results one can fix the t_c -values by the help of FESR (local duality) [27,16]:

$$\begin{aligned} \mathcal{M}_n^{(1,0)} &= \int_0^{t_c} dt t^n \frac{1}{\pi} \text{Im}\Pi_{V/A}^{(1,0)}(t) , \\ R_n(t_c) &\equiv \frac{\int_0^{t_c} dt t^{n+1} \frac{1}{\pi} \text{Im}\Pi_{V/A}^{(1,0)}(t)}{\int_0^{t_c} dt t^n \frac{1}{\pi} \text{Im}\Pi_{V/A}^{(1,0)}(t)} \simeq M_H^2 . \end{aligned} \quad (11)$$

The results discussed in the next section will satisfy these stability criteria.

3. The hybrid two-point function in QCD

A QCD analysis of the two-point function have been done in the past by different groups [4,5], where (unfortunately) the non-trivial QCD expressions were wrong

leading to some controversial predictions [7]. In this paper, we extend the analysis by taking into account the non-trivial α_s correction and the effect of the new $1/q^2$ term not taken into account into the SVZ expansion. The corrected QCD expressions of the correlator are given in [6,9,7] to lowest order of perturbative QCD but including the contributions of the condensates of dimensions lower than or equal to six. The new terms appearing in the OPE are presented in the following ²:

- The perturbative QCD expression including the NLO radiative corrections reads [11]:

$$\begin{aligned}\text{Im}\Pi_{V/A}^{(1)}|_{\text{pert}} &= \frac{\alpha_s}{60\pi^2}t^2 \left[1 + \frac{\alpha_s}{\pi} \left[\frac{121}{16} \right. \right. \\ &\quad \left. \left. - \frac{257}{360}n_f + \left(\frac{35}{36} - \frac{n_f}{6} \right) \log \frac{\nu^2}{t} \right] \right] \\ \text{Im}\Pi_{V/A}^{(0)}|_{\text{pert}} &= \frac{\alpha_s}{120\pi^2}t^2 \left[1 + \frac{\alpha_s}{\pi} \left[\frac{1997}{432} - \frac{167}{360}n_f \right. \right. \\ &\quad \left. \left. + \left(\frac{35}{36} - \frac{n_f}{6} \right) \log \frac{\nu^2}{t} \right] \right] \quad (12)\end{aligned}$$

- The anomalous dimension of the current can be easily deduced to be [11]:

$$\gamma \equiv \nu \frac{d}{d\nu} \mathcal{O}_V^\mu = \left[\gamma_1 \equiv -\frac{16}{9} \right] \frac{\alpha_s}{\pi} \mathcal{O}_V^\mu. \quad (13)$$

- The lowest order correction due to the (short distance) tachyonic gluon mass reads:

$$\begin{aligned}\text{Im}\Pi_{V/A}^{(1)}(t)_\lambda &= -\frac{\alpha_s}{60\pi^2} \frac{35}{4} \lambda^2 t \\ \text{Im}\Pi_{V/A}^{(0)}(t)_\lambda &= \frac{\alpha_s}{120\pi^2} \frac{15}{2} \lambda^2 t \quad (14)\end{aligned}$$

- The contributions of the dimension-four condensates reads in the limit $m^2 = 0$ to LO [6,9,7] and to NLO [12]:

$$\begin{aligned}\text{Im}\Pi_V^{(1)}|_4 &= \frac{1}{9\pi} \left[\alpha_s \langle G^2 \rangle \left[1 - \frac{145}{72} \frac{\alpha_s}{\pi} \right. \right. \\ &\quad \left. \left. + \frac{8}{9} \frac{\alpha_s}{\pi} \log \frac{t}{\nu^2} \right] + 8\alpha_s m \langle \bar{\psi}\psi \rangle \right] \\ \text{Im}\Pi_A^{(0)}|_4 &= -\frac{1}{6\pi} \left[\alpha_s \langle G^2 \rangle \left[1 - \frac{209}{72} \frac{\alpha_s}{\pi} \right. \right. \\ &\quad \left. \left. + \frac{8}{9} \frac{\alpha_s}{\pi} \log \frac{t}{\nu^2} \right] - 8\alpha_s m \langle \bar{\psi}\psi \rangle \right], \quad (15)\end{aligned}$$

where $a_s \equiv \alpha_s/\pi$ and $\langle \bar{\psi}\psi \rangle \equiv \langle \bar{\psi}_u \psi_u \rangle \simeq \langle \bar{\psi}_d \psi_d \rangle$.

² We neglect some possible mixings with operators containing more γ -matrices like $g\bar{\psi}_i\lambda_a\gamma_\mu\sigma_{\nu\lambda}\psi_j G_a^{\mu\nu}$, which is expected to be small.

- To leading order in α_s , the contributions of the dimension-six gluon and mixed condensates read [9,7]:

$$\begin{aligned}\Pi_V^{(1)}|_6 &= \frac{1}{48\pi^2 q^2} \left[g^3 \langle G^3 \rangle - \frac{83}{9} \alpha_s m g \langle \bar{\psi} G \psi \rangle \right] \\ \Pi_A^{(0)}|_6 &= \frac{11}{18} \frac{\alpha_s}{\pi} \frac{1}{q^2} m g \langle \bar{\psi} G \psi \rangle \log -\frac{q^2}{\nu^2}, \quad (16)\end{aligned}$$

where one can notice the miraculous cancellation of the log-coefficient of the dimension-six condensates for $\Pi_V^{(1)}$.

- The four-quark condensate contributions including radiative corrections read for n flavours [12]:

$$\begin{aligned}\Pi_V^{(1)}|_6 &= \frac{1}{q^2} \frac{16}{9} \alpha_s \langle \bar{\psi}\psi \rangle^2 \left[1 + \frac{1}{18} \left(\frac{91}{6} - 5n_f \right) \frac{\alpha_s}{\pi} \right. \\ &\quad \left. + \frac{1}{6} \left(\frac{11}{12} + n \right) \frac{\alpha_s}{\pi} \log -\frac{q^2}{\nu^2} \right]. \quad (17)\end{aligned}$$

- The contribution of the four-quark condensate in the (pseudo)scalar channels vanishes to leading order in α_s and starts at order $\alpha_s^2 \langle \bar{\psi}\psi \rangle^2$. In the scalar channel, it reads [12]:

$$\begin{aligned}\Pi_V^{(0)}|_6 &= \frac{1}{q^2} \frac{16}{3} \alpha_s \langle \bar{\psi}\psi \rangle^2 \left[\frac{1}{3} \left(\frac{14}{3} + \frac{n_f}{2} \right) \frac{\alpha_s}{\pi} \right. \\ &\quad \left. - \frac{1}{12} \left(\frac{53}{12} + n_f \right) \frac{\alpha_s}{\pi} \log -\frac{q^2}{\nu^2} \right]. \quad (18)\end{aligned}$$

4. Upper bound on $M_H(1^{+-})$ from LSR

Using the positivity of the spectral function, we can deduce an upper bound on M_H from $\mathcal{R}_n(\tau)$: $n = 0, 1$. We show the result in Fig. 1 for the 1^{+-} channel. The result is given in the domain limited by the two full (red) curves from $\mathcal{R}_0(\tau)$ and by the two green curves (dashed) from $\mathcal{R}_1(\tau)$ spanned by the two extremal values of the set of QCD parameters given in Table 1.

Table 1

Value of the upper bound on M_H in GeV at the minima of $\mathcal{R}_0(\tau)$ and at its intersection with $\mathcal{R}_1(\tau)$ for two extremal values of the Sets of Power Corrections in units of GeV^d (d is their dimension).

Power Corrections	$(\alpha_s/\pi)\lambda^2$	$\alpha_s \langle G^2 \rangle$	$\alpha_s \langle \bar{\psi}\psi \rangle^2 \times 10^4$	M_{π_1}
Set 1	-0.04	0.055	4.8	≤ 2.0
Set 2	-0.10	0.081	4.2	≤ 2.4

One can note that the prediction increases by 100 MeV each when $|\lambda^2|$ and $\alpha_s \langle G^2 \rangle$ decrease from their central

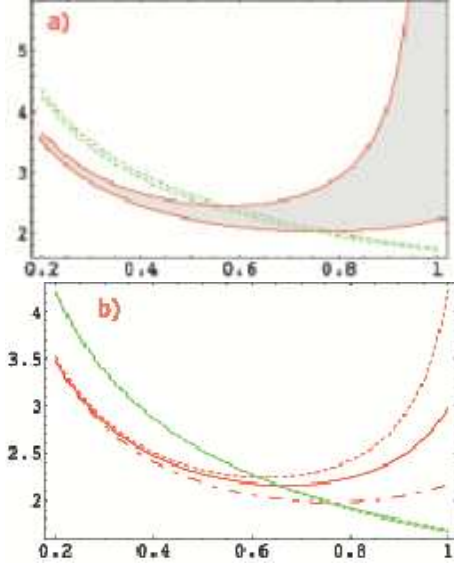


Figure 1. a) Domain of upper bound on M_H in GeV versus the LSR variable τ in GeV^{-2} for two extremal values of the Sets of Power Corrections in Table 1 from $\mathcal{R}_0(\tau)$ (red: continuous) and $\mathcal{R}_1(\tau)$ (green: dashed); b) Effect of radiative corrections on the central value of the bound of M_H from $\mathcal{R}_0(\tau)$ red: : LO (dashed-dotted); NLO condensate+LO perturbative (dashed) ; Total NLO (continuous). Green curves are from $\mathcal{R}_1(\tau)$.

value to the one allowed in the range given in Section 2, while it increases by 25 MeV when $10^4 \alpha_s \langle \bar{\psi}\psi \rangle^2$ increases from 4.5 to 4.8. The effects of the variation of Λ and M_0^2 within the range in Section 2 are invisible. At the stability points (in the sum rule variable τ) of the continuous (red) curves which are also the intersections with the dashed (green) curves, one can deduce:

$$M_H(1^{++}) \leq (2.2 \pm 0.2) \text{ GeV} . \quad (19)$$

This value confirms and improves previous results in [9,11,12]. The result $(2.2 \pm 0.2) \text{ GeV}$ from lattice with two dynamical quarks though still consistent with this bound is on its boarder. We show in Fig 1 the effects of radiative corrections on the results. One can see that the ones due to the condensates give larger effects and increase the bound from 2.0 (LO) to 2.25 GeV, while the ones due to the perturbative terms decrease the mass prediction by 0.05 MeV, where the α_s -correction and the one induced by the anomalous dimension act in the opposite directions. The ratio \mathcal{R}_1 is almost unaffected by the radiative corrections [dotted (green) curves on top of each others].

5. QSSR predictions of f_H and M_H for 1^{++}

Following Ref. [14], we introduce the RGI coupling \hat{f}_H defined as:

$$f_H(\nu) = \frac{\hat{f}_H}{(\log \nu / \Lambda)^{\gamma_1 / -\beta_1}} , \quad (20)$$

with γ_1 the anomalous dimension in Eq. (13) and $-\beta_1 = 1/2(11 - 2n_f/3)$ the first coefficient of the β function for n_f flavours.

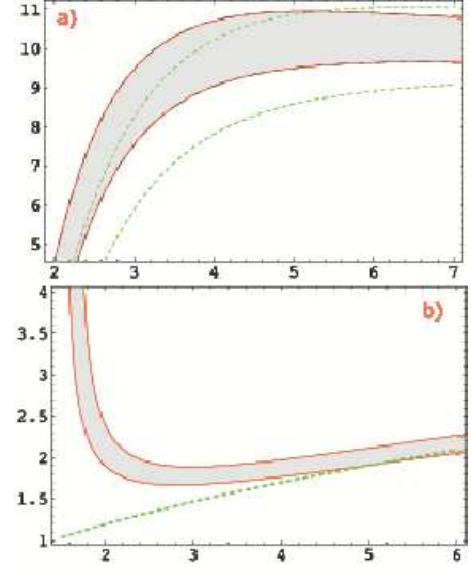


Figure 2. a) Domain of FESR predictions versus the QCD continuum threshold t_c using the set of power correction in Table 1 for f_H from \mathcal{M}_0 (red: continuous) and \mathcal{M}_1 (black: dashed (on top of the red curve)) and \mathcal{M}_2 (green: dotted); b) The same as for f_H but for M_H : from $R_0(t_c)$ (red: continuous); from $R_1(t_c)$ (green: dotted).

Using FESR, we show the predictions for the decay constant in Fig. 2a) and the mass of the 1^{++} in in Fig. 2b). Using the stability criterion on the variation of t_c , and the intersection of the predictions of different moments, and the Sets of Power Corrections in Table 1, one obtains the optimal value for the 1^{++} :

$$\begin{aligned} \hat{f}_H &= 9.6(1.4) \text{ MeV} : t_c = (4.4 \sim 5.2) \text{ GeV}^2 \\ M_H &= 1.80(10) \text{ GeV} : t_c = (2.8 \sim 5.0) \text{ GeV}^2. \end{aligned} \quad (21)$$

We show in Fig. 3 the predictions of the LSR in the 1^{++} channel using the central values of the QCD input parameters and using two values of $t_c = 4.6$ (beginning of the τ -stability) and 5 GeV^2 fixed from the previous FESR results. The prediction for the decay constant is not conclusive as there is not a stability in the LSR variable τ . However, one can see from Fig. 3 that the different predictions interact for :

$$\hat{f}_H = (10 \sim 22) \text{ MeV} , \quad (22)$$

which is consistent with the previous FESR result but less accurate. Therefore, we consider as a final prediction the one from FESR. For the mass prediction, the LSR gives:

$$M_H = 1.81(3)(7) \text{ GeV} : t_c = (4.6 \sim 5.0) \text{ GeV}^2, \quad (23)$$

where the 1st and 2nd errors come respectively from t_c and the QCD input parameters. We take the (naïve

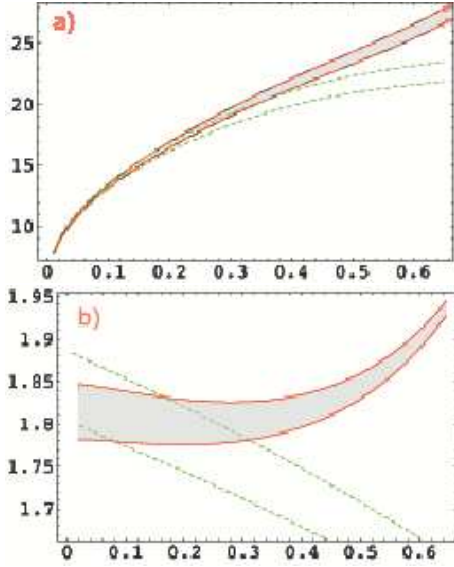


Figure 3. Predictions of the LSR in the 1^{-+} hybrid channel using the central values of the QCD input parameters and two values of $t_c = 4.6$ (beginning of the τ -stability) and 5 GeV^2 fixed from the previous FESR results: a) RG invariant coupling constant f_H from \mathcal{L}_0 (red: continuous), from \mathcal{L}_1 (black: dashed-dotted (on top of the red curve)), from \mathcal{L}_1 (green: dashed); b) M_H from $\mathcal{R}_0(\tau)$ (red: continuous) and from $\mathcal{R}_1(\tau)$ (green: dotted).

arithmetic) average of the FESR and LSR results and we take the quadratic average of the errors. Then, we obtain:

$$\langle M_H \rangle = 1.81(6) \text{ GeV} . \quad (24)$$

6. Comparison with existing theoretical results

The central value of the result in Eq. (24) has increased by 100 – 200 MeV compared to the previous QSSR results quoted in Eq. (3). The difference with the one in Ref. [9,11] is due to the inclusion of radiative corrections for both perturbative and power corrections here, which increase the mass prediction by about 200 MeV. The difference with Ref. [12] is the non-inclusion of the tachyonic gluon mass and the use of factorization for the four-quark condensate in [12]. In [13], a value of 1.2 GeV for the mass has been obtained³, which the author attributes to be due to the hybrid operator anomalous dimension. In this paper, we show that the effect of perturbative radiative corrections including the one due to the anomalous dimension only decreases the mass predictions by 50 MeV. This result is (a priori) expected where it is easy to show that perturbative radiative corrections tend to cancel in the ratio of moments used to extract the meson mass. A similar explicit exam-

³Also, in [13] an operator having high number of γ matrices has been considered in the light-cone gauge but its correspondence in a covariant gauge is not quite transparent. The current looks to be a tensor current while from the definition of the matrix element, the 1^{-+} hybrid meson contributes through its longitudinal (i.e. spin zero) part.

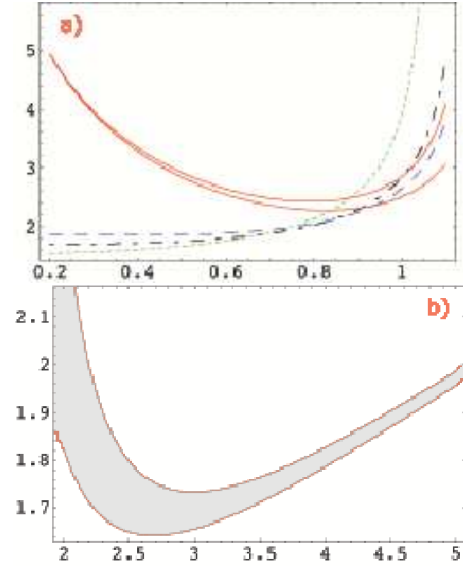


Figure 4. Predictions of the four-quark state mass in GeV using the current in Eq. (26): a) LSR: domain spanned by the upper bound versus τ using the largest range spanned by the QCD parameters given after Eq. (7) (red: continuous curve); Mass using the central values of QCD parameters for $t_c = 3 \text{ GeV}^2$ (green: dashed), $t_c = 4 \text{ GeV}^2$ (black: dotted-dashed), $t_c = 5 \text{ GeV}^2$ (blue: dashed). b) FESR versus t_c : region spanned by the largest range of QCD parameters.

ple has been studied for the case of $2^{++}\bar{q}q$ current [14]. The present result is slightly lower than the lattice results [2,3] quoted in Eq. (1). As the α_s corrections are reasonably small, we do not see in the QCD side any potential contributions which can restore the discrepancy between QSSR and lattice results. From the phenomenological side, a more involved parametrization of the spectral function might help, but in many known channels, the usual duality ansatz : one resonance + QCD continuum describes accurately the spectral function at the t_c and τ stability points even for broad states like e.g. the σ and K_0^* mesons [33].

7. Masses of the 1^{-+} four-quark states

We reconsider the QSSR analysis in [22] using the previous set of QCD parameters where the values of the gluon and four-quark condensates are about a factor 2 higher here. We shall introduce the renormalization group invariant $\langle \bar{\psi}\psi \rangle$ condensate [34]:

$$\hat{\mu}_i^3 = \frac{\langle \bar{\psi}_i \psi_i \rangle(\nu)}{(\log \nu / \Lambda)^{2/\beta_1}} , \quad (25)$$

with [35]: $\hat{\mu}_u = -(263 \pm 7) \text{ MeV}$. We neglect the small Q^2 -dependence of $\alpha_s \langle \bar{\psi}\psi \rangle$ and $\langle g\bar{\psi}G\psi \rangle$ as well as the anomalous dimension of the four-quark current, which should have small effect in the mass determination. The four-quark meson can be described by the diquark anti-

diquark operators ⁴:

$$\begin{aligned}\eta_{1\mu}^M &= u_a^T C \gamma_\mu d_b (\bar{u}_a C \bar{d}_b^T + \bar{u}_b C \bar{d}_a^T) + \dots \\ \eta_{3\mu}^M &= u_a^T C d_b (\bar{u}_a \gamma_\mu C \bar{d}_b^T - \bar{u}_b \gamma_\mu C \bar{d}_a^T) + \dots\end{aligned}\quad (26)$$

where ... denotes an interchange between γ_u and 1 or γ_5 . The QCD expression of the LSR of the corresponding correlator can be written to LO in α_s [22]:

$$\begin{aligned}\mathcal{L}^{4q}(\tau) &= \int_0^{\tau_c} dt (c_0 t^4 + c_4 t^2 + c_6 t + c_8) e^{-t\tau} \\ &\quad + c_{10} + c_{12}\tau\end{aligned}\quad (27)$$

where, for $\eta_{1\mu}^M$:

$$\begin{aligned}c_0 &= \frac{1}{18432\pi^6}, \quad c_4 = -\frac{\langle g^2 G^2 \rangle}{18432\pi^6}, \\ c_6 &= \frac{\langle \bar{\psi}\psi \rangle^2}{18\pi^2}, \quad c_8 = -\frac{\langle \bar{\psi}\psi \rangle \langle g\bar{\psi}G\psi \rangle}{24\pi^2}, \\ c_{10} &= \frac{\langle g\bar{\psi}G\psi \rangle}{192\pi^2} - \frac{5}{864\pi^2} \langle g^2 G^2 \rangle \langle \bar{\psi}\psi \rangle^2, \\ c_{12} &= -\frac{32}{81} g^2 \langle \bar{\psi}\psi \rangle^4 + \frac{\langle g^2 G^2 \rangle \langle \bar{\psi}\psi \rangle \langle g\bar{\psi}G\psi \rangle}{576\pi^2},\end{aligned}\quad (28)$$

from which one can deduce the ratio $\mathcal{R}_n(\tau)$ and $R_n(t_c)$ defined in Eqs. (10) and (11) used for extracting the meson mass. The result of the analysis is shown in Fig. 4 from which we can deduce in units of GeV:

$$M_{4q1} \leq 2.4(1) : \text{LSR} ; M_{4q1} = 1.70(4) : \text{FESR}, \quad (29)$$

for $t_c \simeq (2.5 \sim 3) \text{ GeV}^2$.

where one can notice that the LSR result does not have τ - and t_c -stabilities such that only an upper bound can be extracted from the positivity of the spectral function. This result is slightly higher than the one in [22] due to the consideration of the violation of factorization in the estimate of the high-dimension condensates here. We have repeated the analysis for the $\eta_{3\mu}^M$. We show in Fig 5a) the result of the analysis from the LSR moment $\mathcal{R}_0(\tau)$ which presents τ -stability for $t_c \geq 4 \text{ GeV}^2$ but no t_c -stability. In the expanded form, FESR $R_0(t_c)$ presents a zero for $2.5 \leq t_c \leq 7 \text{ GeV}^2$ due to $\mathcal{M}_0(t_c)$ signaling large non-perturbative effects for this quantity. This zero shows up like a minimum in t_c in the non-expanded form of $R_0(t_c)$ given a mass $M \approx 1.2 \text{ GeV}$, but at too low value of $t_c \approx 2. \text{ GeV}^2$, where the OPE does not converge ⁵. At this t_c -values, the LSR does not also show a τ -stability. Therefore, we do not retain this result from our analysis. $R_1(t_c)$ moment has a much better behaviour but (unfortunately) does not show a t_c -stability [see Fig. 5b)]. The most conservative result from both LSR and FESR is:

$$1.64 \leq M_{4q3} \leq 2.1(1) : t_c \geq 4 \text{ GeV}^2. \quad (30)$$

⁴In principle, these operators should mix under renormalizations [36] and one should built their renormalization group invariant (RGI) combination for describing the physical state.

⁵In [22], this minimum corresponds to a slightly higher value of the mass due to the different choice of the QCD set of parameters.

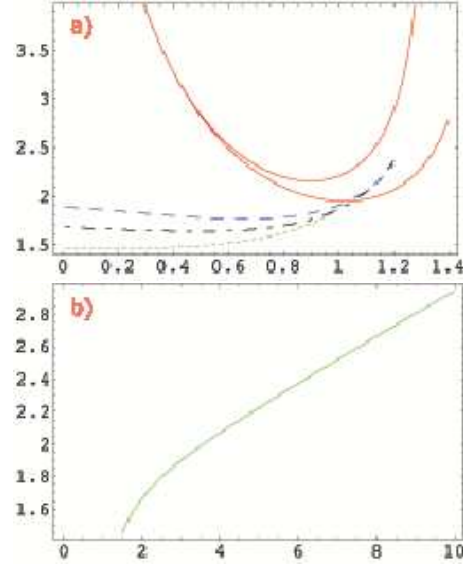


Figure 5. Predictions of the four-quark state mass in GeV using the current in Eq. (31): a) LSR: domain spanned by the upper bound versus τ using the largest range spanned by the QCD parameters given after Eq. (7) (red: continuous curve); Mass using the central values of QCD parameters for $t_c = 3 \text{ GeV}^2$ (green: dashed), $t_c = 4 \text{ GeV}^2$ (black: dotted-dashed), $t_c = 5 \text{ GeV}^2$ (blue: dashed). b) FESR versus t_c using the central values of QCD parameters.

where the upper bound comes from the positivity of the LSR moment. Currents with higher number of γ matrices have been also considered in [22]:

$$\eta_{2\mu,4\mu}^M \equiv u_a^T C \gamma^\nu \gamma_5 d_b (\bar{u}_a C \sigma_{\mu\nu} \gamma_5 \bar{d}_b^T \pm \bar{u}_b C \sigma_{\mu\nu} \gamma_5 \bar{d}_a^T + \dots), \quad (31)$$

where ... is an interchange between γ^ν and $\sigma_{\mu\nu}$. The analysis of the corresponding correlators lead to similar results (within the errors) than the ones in Eq. (26):

$$M_{\eta_{4\mu}} \approx M_{\eta_{1\mu}} \quad \text{and} \quad M_{\eta_{3\mu}} \approx M_{\eta_{2\mu}}. \quad (32)$$

For definiteness, we consider that we have only one 4-quark state coupled to a RGI operator which results from a combination of these different operators expected to mix under renormalizations [36]. We fix the four-quark state mass to the value from Eq. (29):

$$M_{4q} = 1.70(4) \text{ GeV}. \quad (33)$$

8. Masses of the 1^{-+} molecules

The possibility to form two 1^{-+} molecules with the two operators:

$$\begin{aligned}J_1^\mu &\sim (\bar{\psi}\gamma_5\psi)(\bar{\psi}\gamma_5\gamma^\mu\psi), \\ J_2^{\mu\nu} &\sim \epsilon^{\nu\rho\sigma} [(\bar{u}\gamma_5\gamma_\rho d)(\bar{d}\gamma^\sigma u) - (\bar{d}\gamma_5\gamma_\rho u)(\bar{u}\gamma^\sigma d)]\end{aligned}\quad (34)$$

has been discussed in [23], where the meson associated to the 1st (resp 2nd) operator is expected to decay into $\eta\pi$, $\eta'\pi$ (resp. $\rho\pi$, $b_1\pi$). Using the QCD expression given in [23] without the one-direct instanton contribution which is largely affected by the uncertainty of the

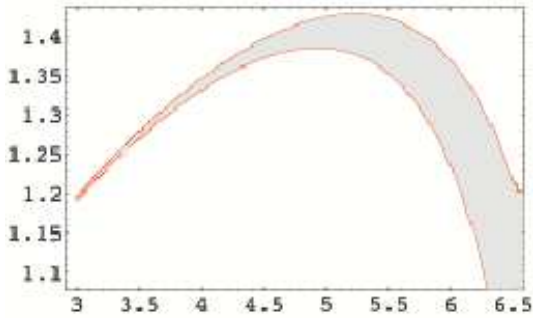


Figure 6. Mass of the molecule state associated to the current J_1^μ in Eq. (34) versus t_c using the largest range spanned by the QCD parameters given after Eq. (7) (red: continuous curve).

instanton density ρ appearing as ρ^6 , we use the lowest FESR moment $R_0(t_c)$ for the analysis of the meson mass M_{mol1} associated to the current J_1^μ where a t_c -stability is obtained at $t_c \simeq 5 \text{ GeV}^2$ (Fig. 6), while the LSR moments $\mathcal{R}_{0,1,2}$ do not give conclusive results. The mass M_{mol2} associated to the current J_2^μ increases with t_c for the FESR, while the LSR moments decrease slowly with the sum rule variable τ but present a zero around $\tau = 0.7 - 0.8 \text{ GeV}^{-2}$ (not shown in Fig. 7), while it presents t_c -stability around 3 GeV^2 . The optimal results are:

$$M_{mol1} = 1.41(3) \text{ GeV}, \quad M_{mol2} \simeq (1.2 \sim 1.4) \text{ GeV}, \quad (35)$$

which agree within the errors with the ones in [23]. However, we can consider that the two operators in Eq. (34) mix under renormalization such that only one physical state couples to the corresponding RGI operator, which we fix to be:

$$M_{mol} = 1.3(1) \text{ GeV}. \quad (36)$$

9. Nature of the $\pi_1(1400)$, $\pi_1(1600)$ and $\pi_1(2015)$

From your previous analysis and for a further phenomenological use, we shall consider one hybrid, one four-quark and one molecule states below 2 GeV , with the masses from Eqs. (24), (33) and (36) in GeV :

$$M_H = 1.81(6), \quad M_{4q} = 1.70(4), \quad M_{mol} = 1.3(1). \quad (37)$$

The $\pi_1(1400)$ is intriguing as it is seen to decay into $\eta\pi$ and $\eta'\pi$ but not into $\rho\pi$, $b_1\pi$ [1]⁶. The later decays being also expected for an hybrid state [9]. Looking at our *bare (unmixed)* mass predictions in Eq (37), one may expect that the molecule state is the most probable candidate. We consider that the suppression of the $\rho\pi$ decay can be due to a mixing of this molecular state with

⁶For a review on different experimental results and related problems, see e.g. [37].

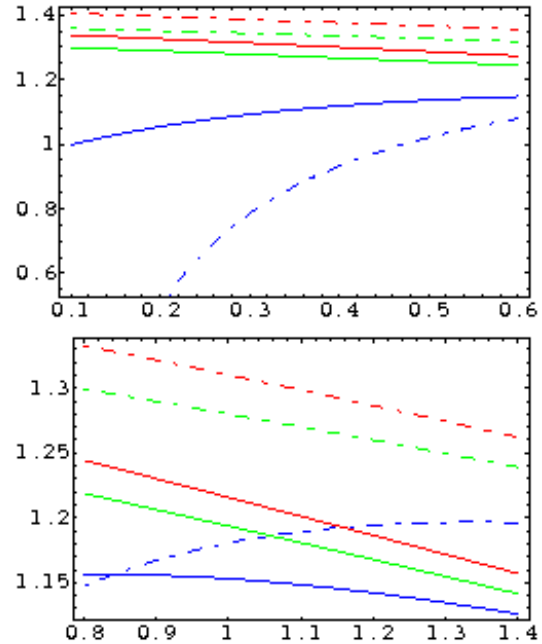


Figure 7. Mass of the molecule state associated to the current J_2^μ in Eq. (34) from LSR versus the sum rule variable $\tau = 0.1$ to 0.6 GeV^{-2} and 0.8 to 1.4 GeV^{-2} and for different values of t_c using the central values of the QCD parameters given after Eq. (7) and for two LSR moments \mathcal{R}_0 (continuous curve) and \mathcal{R}_1 (dotted-dashed curve): $t_c = 2.5 \text{ GeV}^2$ (green), $t_c = 3 \text{ GeV}^2$ (red) and $t_c = 3.5 \text{ GeV}^2$ (blue).

the four-quark or/and hybrid states. We use a minimal two-component mixing:

$$\begin{aligned} \pi_1(1400) &= \cos \theta_{mol} |mol\rangle + \sin \theta_{mol} |X\rangle \\ \pi_1(1600) &= -\sin \theta_{mol} |mol\rangle + \cos \theta_{mol} |X\rangle, \end{aligned} \quad (38)$$

where X is a four-quark or hybrid state. The “best fit” is obtained for the sets :

$$\begin{aligned} (M_{mol}, M_X) &= (1.2 \sim 1.3; 1.70, 1.74) \text{ GeV} \implies \\ \theta_{mol} &\simeq -(11.7 \pm 2.2)^\circ, \end{aligned} \quad (39)$$

which is slightly favours a molecule/four-quark mixing. This result may suggest that the $\pi_1(2015)$, quoted in the extended version of PDG [1], can have more hybrid state in its wave function. One also expects that the non-exotic meson 1^{--} as well as the isospin partners of these 1^{-+} exotics should be almost degenerate in masses with these exotic states, while some of their radial excitations have masses of the order of optimal continuum threshold:

$$M' \approx \sqrt{t_c} \simeq (1.7 \sim 2.2) \text{ GeV}. \quad (40)$$

Further experimental and theoretical tests of this mixing scheme are required.

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